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A Novel Theoretical Framework Formulated for Information Discovery from Number System and Collatz Conjecture Data

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Abstract

Newly discovered fundamental theories (metamathematics) of integer numbers may be used to formalise and formulate a new theoretical number system from which other formal analytical frameworks may be discovered, primed and developed. The proposed number system, as well as its most general framework which is based on the modelling results derived from an investigation of the Collatz conjecture (i.e., the 3x+1 problem), has emerged as an effective exploratory tool for visualising, mining and extracting new knowledge about quite a number of mathematical theorems and conjectures, including the Collatz conjecture. Here, we introduce and demonstrate many known applications of this prime framework and show the subsequent results of further analyses as new evidences to justify the claimed fascinating capabilities of the proposed framework in computational mathematics, including number theory and discrete mathematics.

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Keywords: formal system; Collatz conjecture; 3n+1 challenge, stopping time algorithm; congruence classes, Collatz map, system theory; model theory; number theory; discrete mathematics; modelling; integer classification

1. Introduction

Define S the set of all positive numbers $N = \{1, 2, 3, 4, \dots\}$ and $f(n)$ the Collatz mapping operation

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2} \end{cases}$$

performed on any input n where n is a positive integer chosen from N [1]. This process may be viewed as relating certain odd integers to certain other odd integers through some certain even integers. The Collatz conjecture (CC) claims that repeated iteration of the Collatz mapping operation on n will eventually reach the number 1, regardless of which positive number n is chosen initially, according to the mathematician Lothar Collatz who first proposed the

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conjecture in 1987. The stopping time of n [1], often denoted as $\sigma_\infty(n)$, is the total number of iterations required to reach the number 1 [2, 3]. The CC [4] is known by many other names, e.g the 3x+1 conjecture [5], the (3n+1)-problem [6, 7], the Ulam conjecture, the Thwaites conjecture, the Kakutani's problem, the Hasse's algorithm, and the Syracuse problem [8, 9, 10]. It is considered by some as an important and exceptionally rich mathematical research [11].

Though the CC challenge is easy to state, the general opinion shared by many mathematicians is that the problem is extremely difficult to prove [12, 13] because of its unpredictable iterations [14]. Many leading experts in the field have investigated and written articles about the conjecture [12, 14, 15]. We recommend the annotated bibliographies [2, 16, 17] for a more comprehensive study about the background of the CC challenge. The CC may also be viewed as a string rewriting system [15].

Here a novel representation of the Collatz operations is captured and demonstrated in Fig. 1 for the first few integers and general sets of numbers.

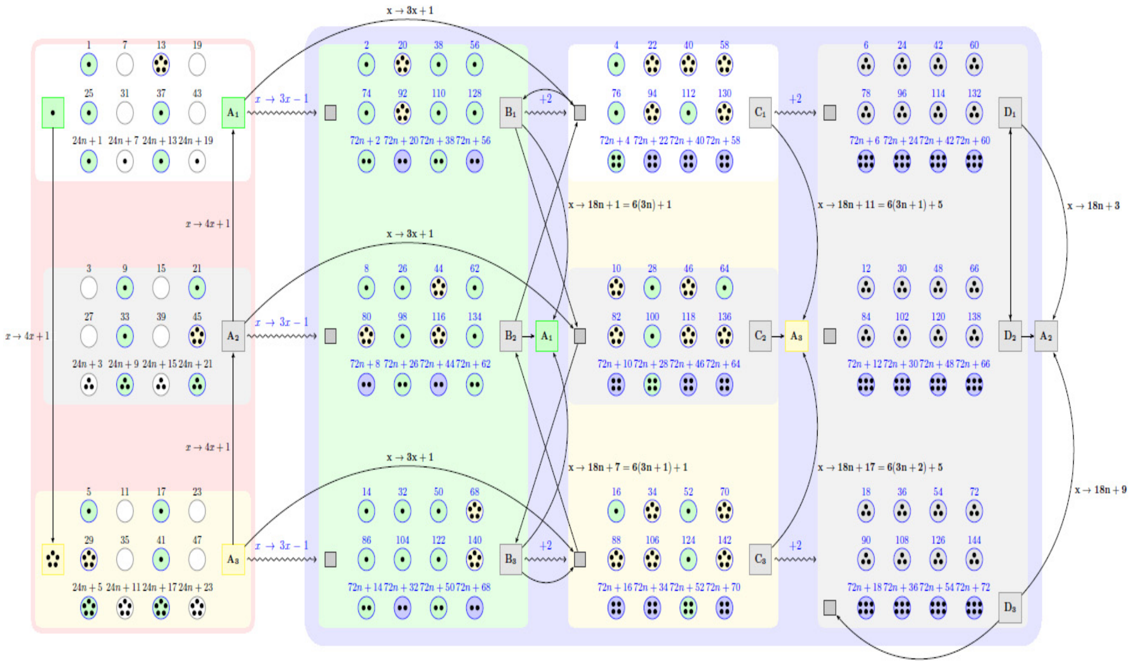

Fig.1. A novel Collatz map constructed for investigating the dynamics of the Collatz based number system. An initial analysis of this Collatz map created and designed by the author indicates the relations between odd number inputs, intermediate even numbers and nature of their next odd number destinations in terms of $[1]_6$, $[3]_6$ and $[5]_6$ subsequences.

The principal aim of this paper is to systematically formulate, further develop and present a novel (theoretical) framework of the Collatz system in the form of a novel classification of the number system. It is believed that a further (re)classification of the initial classified (multilevel) number system presented in [18, 19] may result in uncovering new insights and further inspire new ideas about how to tackle the mathematical mysteries surrounding the (un)decidability of the problem.

Nomenclature

CC	the Collatz Conjecture; the *3n+1* challenge
$\sigma_\infty(n)$	the total stopping time – no of iterations required to reach the number 1

1.1. A new Collatz based classification of positive integers

The proposed Collatz based number system map first presented in [18] (here presented in Fig. 2 below) shows a multilevel classification of the positive odd integers in 9 distinct congruence classes modulo 18 based on a simple '*4n+1*' relation between integer numbers. For example, in the figure below (Fig. 3) a related vector of odd integers associated with the '*4n+1*' relation is depicted in red colour to illustrate other similar relationships among the entries (partitions of integer sets). Note that the horizontal arrangement of the integers is strictly based on the associated congruence classes in the order *d* depicted in Fig. 2.

$$d = \begin{cases} d_1 & \text{if } d \equiv 1 \pmod{18}, \text{ e.g. } 1, 349525, \ldots \\ d_2 & \text{if } d \equiv 5 \pmod{18}, \text{ e.g. } 5 \\ d_3 & \text{if } d \equiv 3 \pmod{18}, \text{ e.g. } 21 \\ d_4 & \text{if } d \equiv 13 \pmod{18}, \text{ e.g. } 85 \\ d_5 & \text{if } d \equiv 17 \pmod{18}, \text{ e.g. } 341 \\ d_6 & \text{if } d \equiv 15 \pmod{18}, \text{ e.g. } 1365 \\ d_7 & \text{if } d \equiv 7 \pmod{18}, \text{ e.g. } 5461 \\ d_8 & \text{if } d \equiv 11 \pmod{18}, \text{ e.g. } 21845 \\ d_9 & \text{if } d \equiv 9 \pmod{18}, \text{ e.g. } 87381 \\ \hline d_{10} & \text{if } d \equiv 1 \pmod{18}, \text{ e.g. } 349525 : d_{10} = [1]_{18} = d_1 \\ \vdots & \end{cases}$$

Fig.2. The fundamental congruence classes used to create a multilevel classification of integer numbers according to the proposed '*4n+1*' rule.

To create a new multilevel classification (layout) of odd integers in relation to their next odd number sequences the following set of numbers emerges which may be further subdivided into many level of layers, namely: layer 1 (top-most e.g. *36n+19*); layer 2 (second e.g. *72n+1*), and so forth; each layer having a '3n+1' relation to a set of certain even numbers (and other subsequences of even numbers) before reaching the next odd numbers of the Collatz sequence.

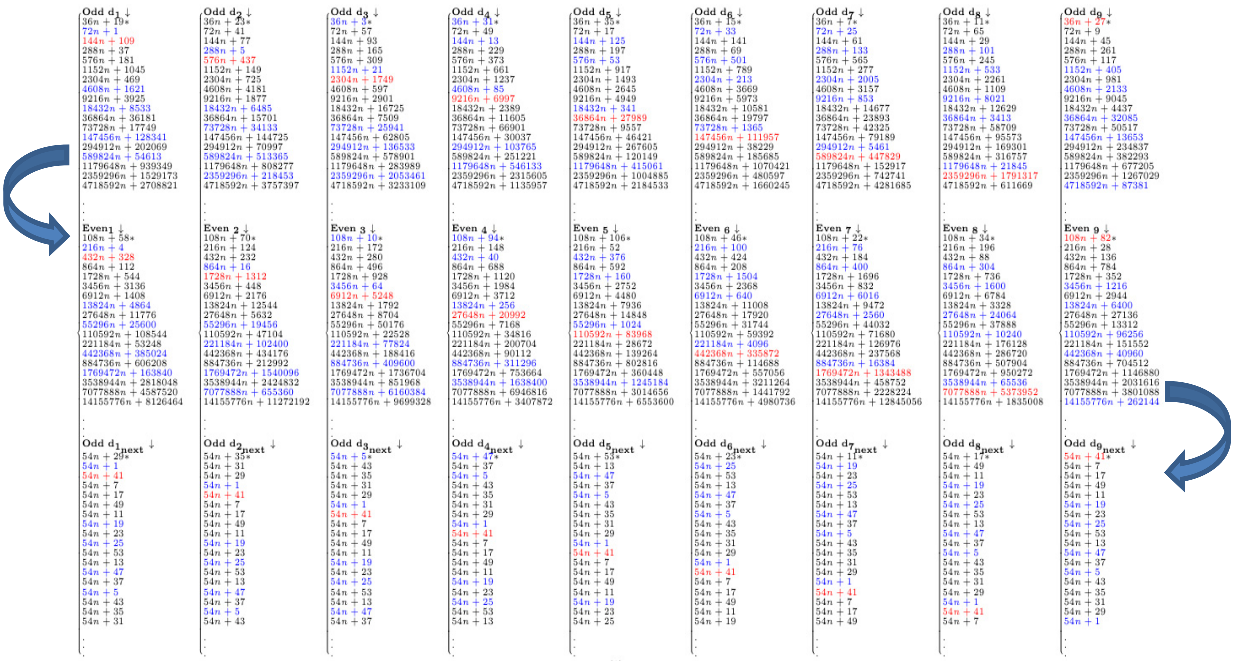

Fig.3. A new classification (layout) of odd integers according to the next odd integers in the $(3n+1)/2^m$ sequence and certain *4n+1* relation of the (associated) diagonals; m is the associated layer number.

1.2. A new stopping time inference strategy

The figure above (Fig. 3) enables a new calculation (i.e. novel inference method) of stopping time(s) to be devised based on their relation with other stopping time values of related odd integers within the associated *Odd-NextOdd transitions*. The following figure (fig. 4) illustrates the associated stopping time(s) in relation to the above-listed numbers.

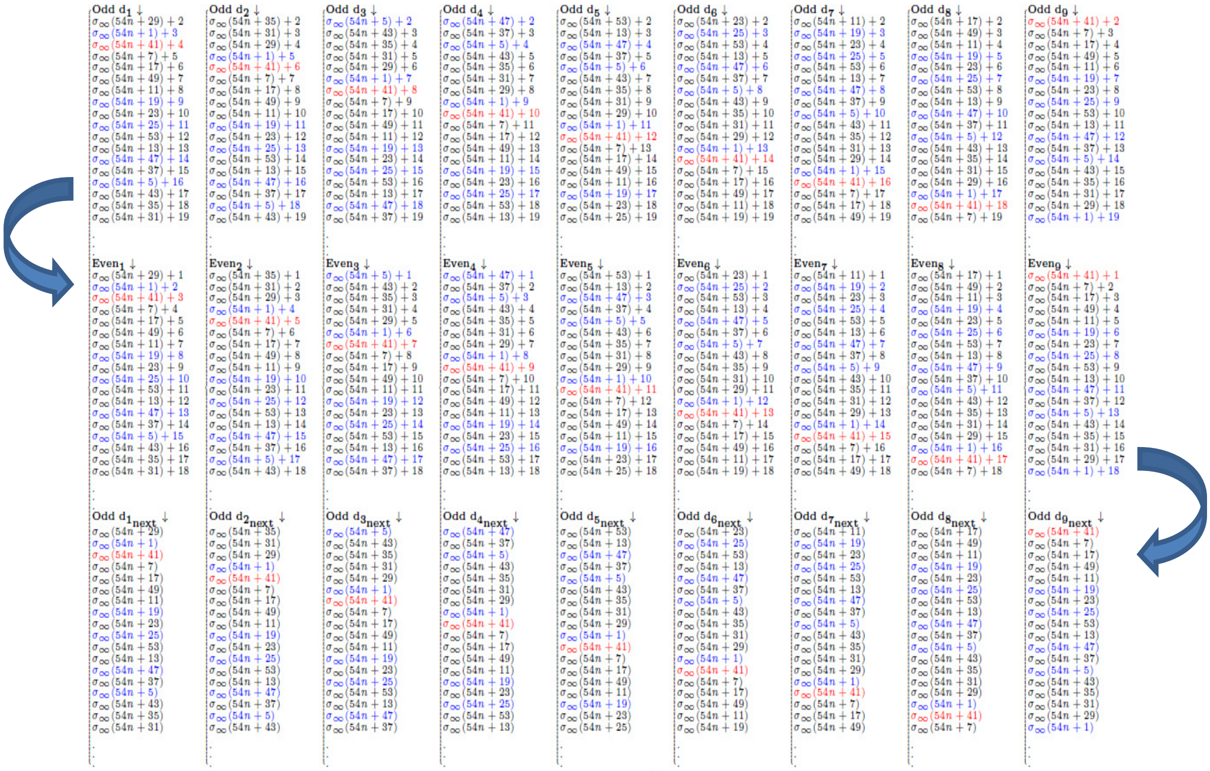

Fig.4. A layout of the associated stopping times of the integers listed in figure 2

Ultimately the technique enables a new (re)formulation of odd integers according to the observed patterns identified and revealed (the RHS of figure 5) below; the whole set of odd number *S* is divided into many integer partitions, i.e. $S_1, S_2, S_3, \ldots$.

$$
S \in 2\mathbb{N}+1 \quad \left|\quad
\begin{array}{l}
S_1 \in 4\mathbb{N}+3, \\
S_2 \in 8\mathbb{N}+1, \\
S_3 \in 16\mathbb{N}+13, \\
S_4 \in 32\mathbb{N}+5, \\
S_5 \in 64\mathbb{N}+53, \\
S_6 \in 128\mathbb{N}+21, \\
S_7 \in 2^8\mathbb{N}+213, \\
S_8 \in 2^9\mathbb{N}+85, \\
S_9 \in 2^{10}\mathbb{N}+853, \\
S_{10} \in 2^{11}\mathbb{N}+341, \\
S_{11} \in 2^{12}\mathbb{N}+3413, \\
S_{12} \in 2^{13}\mathbb{N}+1365, \\
S_{13} \in 2^{14}\mathbb{N}+13653, \\
S_{14} \in 2^{15}\mathbb{N}+5461, \\
S_{15} \in 2^{16}\mathbb{N}+54613, \\
S_{16} \in 2^{17}\mathbb{N}+21845, \\
S_{17} \in 2^{18}\mathbb{N}+218453, \\
S_{18} \in 2^{19}\mathbb{N}+87381, \\
\vdots
\end{array}\right.
$$

Fig.5. A new classification (partitioning) of all odd numbers according to a '$3n+1$-result-divisible-by-2^m' characteristic, i.e. their $(3n+1)/2^m$ disability features, e.g. this figure suggests $(3S_1+1)$ is divisible by 2^1, $(3S_2+1)$ is divisible by 2^2, … , $(3S_m+1)$ is divisible by 2^m and so on.

One fundamental implication of the revealed dynamics (Fig. 5) is that in order to prove the Collatz conjecture an essential requirement is to show that all S_1 numbers converge or that the stopping times of all S_1 numbers are all finite. Next we explain a more important application of Fig. 3 that relates to the creation of a multilevel number-dynamics for the investigation of the truth of the conjecture.

2. Inference and Mapping of Multilevel Dynamics Involving Odd to NextOdd to NextNextOdd Transitions

An important application of the findings above, particularly Fig. 3, is the determination of the general equivalence relations between all odd numbers d congruent to mod 36, $[d]_{36}$ integers, and the equivalence odd numbers e congruent to mod 54, $[e]_{54}$ integers – a necessary and important requirement for the mapping and determination of further Collatz subsequences after the first transition to the next odd integers; the next phase of further transitions from the resultant *54n+e* numbers requires casting and rewriting the intermediate *54n+e* results to their related 3*6n+d* equivalence integers with equal magnitudes. The following figure (fig. 6) reports a partial result of converting (casting) to/from $[d]_{36}$ numbers from/to $[e]_{54}$ numbers.

54	mult.	add.	Is Eq.	mult.	add.	Targ	54	mult.	add.	Is Eq.	mult.	add.	Targ
54	$(n)p + 1$	1	36	$3(n) + 1$	19	29	54	$(n)p + 1$	5	36	$3(n) + 1$	23	35
54	$(n)2^{2}$	1	72	$3(n)$	1	1	54	$(n)2^{2} + 2$	5	72	$3(n) + 1$	41	31
54	$(n)2^{3} + 2$	1	144	$3(n)$	109	41	54	$(n)2^{3} + 4$	5	144	$3(n) + 1$	77	29
54	$(n)2^{4} + 6$	1	288	$3(n) + 1$	37	7	54	$(n)2^{4}$	5	288	$3(n)$	5	1
54	$(n)2^{5} + 14$	1	576	$3(n) + 1$	181	17	54	$(n)2^{5} + 8$	5	576	$3(n)$	437	41
54	$(n)2^{6} + 62$	1	1152	$3(n) + 2$	1045	49	54	$(n)2^{6} + 24$	5	1152	$3(n) + 1$	149	7
54	$(n)2^{7} + 94$	1	2304	$3(n) + 2$	469	11	54	$(n)2^{7} + 56$	5	2304	$3(n) + 1$	725	17
54	$(n)2^{8} + 30$	1	4608	$3(n)$	1621	19	54	$(n)2^{8} + 248$	5	4608	$3(n) + 2$	4181	49
54	$(n)2^{9} + 414$	1	9216	$3(n) + 2$	3925	23	54	$(n)2^{9} + 376$	5	9216	$3(n) + 2$	1877	11
54	$(n)2^{10} + 158$	1	18432	$3(n)$	8533	25	54	$(n)2^{10} + 120$	5	18432	$3(n)$	6485	19
54	$(n)2^{11} + 670$	1	36864	$3(n)$	36181	53	54	$(n)2^{11} + 1656$	5	36864	$3(n) + 2$	15701	23
54	$(n)2^{12} + 1694$	1	73728	$3(n) + 1$	17749	13	54	$(n)2^{12} + 632$	5	73728	$3(n)$	34133	25
54	$(n)2^{13} + 7838$	1	147456	$3(n) + 2$	128341	47	54	$(n)2^{13} + 2680$	5	147456	$3(n)$	144725	53
54	$(n)2^{14} + 3742$	1	294912	$3(n)$	202069	37	54	$(n)2^{14} + 6776$	5	294912	$3(n) + 1$	70997	13
54	$(n)2^{15} + 11934$	1	589824	$3(n) + 1$	54613	5	54	$(n)2^{15} + 31352$	5	589824	$3(n) + 2$	513365	47
54	$(n)2^{16} + 61086$	1	1179648	$3(n) + 2$	939349	43	54	$(n)2^{16} + 14968$	5	1179648	$3(n)$	808277	37
54	$(n)2^{17} + 28318$	1	2359296	$3(n)$	1529173	35	54	$(n)2^{17} + 47736$	5	2359296	$3(n) + 1$	218453	5
54	$(n)2^{18} + 224926$	1	4718592	$3(n) + 2$	2708821	31	54	$(n)2^{18} + 244344$	5	4718592	$3(n) + 2$	3757397	43
54	$(n)p$	19	36	$3(n)$	19	29	54	$(n)p$	23	36	$3(n)$	23	35
54	$(n)2^{2} + 1$	19	72	$3(n) + 1$	1	1	54	$(n)2^{2} + 3$	23	72	$3(n) + 2$	41	31
54	$(n)2^{3} + 7$	19	144	$3(n) + 2$	109	41	54	$(n)2^{3} + 1$	23	144	$3(n)$	77	29
54	$(n)2^{4} + 11$	19	288	$3(n) + 2$	37	7	54	$(n)2^{4} + 5$	23	288	$3(n) + 1$	5	1
54	$(n)2^{5} + 3$	19	576	$3(n)$	181	17	54	$(n)2^{5} + 29$	23	576	$3(n) + 2$	437	41
54	$(n)2^{6} + 19$	19	1152	$3(n)$	1045	49	54	$(n)2^{6} + 45$	23	1152	$3(n) + 2$	149	7
54	$(n)2^{7} + 51$	19	2304	$3(n) + 1$	469	11	54	$(n)2^{7} + 13$	23	2304	$3(n)$	725	17
54	$(n)2^{8} + 115$	19	4608	$3(n) + 1$	1621	19	54	$(n)2^{8} + 77$	23	4608	$3(n)$	4181	49
54	$(n)2^{9} + 243$	19	9216	$3(n) + 1$	3925	23	54	$(n)2^{9} + 205$	23	9216	$3(n) + 1$	1877	11
54	$(n)2^{10} + 499$	19	18432	$3(n) + 1$	8533	25	54	$(n)2^{10} + 461$	23	18432	$3(n) + 1$	6485	19
54	$(n)2^{11} + 2035$	19	36864	$3(n) + 2$	36181	53	54	$(n)2^{11} + 973$	23	36864	$3(n) + 1$	15701	23
54	$(n)2^{12} + 3059$	19	73728	$3(n) + 2$	17749	13	54	$(n)2^{12} + 1997$	23	73728	$3(n) + 1$	34133	25
54	$(n)2^{13} + 5107$	19	147456	$3(n) + 1$	128341	47	54	$(n)2^{13} + 8141$	23	147456	$3(n) + 2$	144725	53
54	$(n)2^{14} + 9203$	19	294912	$3(n) + 1$	202069	37	54	$(n)2^{14} + 12237$	23	294912	$3(n) + 2$	70997	13
54	$(n)2^{15} + 1011$	19	589824	$3(n)$	54613	5	54	$(n)2^{15} + 20429$	23	589824	$3(n) + 1$	513365	47
54	$(n)2^{16} + 17395$	19	1179648	$3(n)$	939349	43	54	$(n)2^{16} + 36813$	23	1179648	$3(n) + 1$	808277	37
54	$(n)2^{17} + 115699$	19	2359296	$3(n) + 2$	1529173	35	54	$(n)2^{17} + 4045$	23	2359296	$3(n)$	218453	5
54	$(n)2^{18} + 50163$	19	4718592	$3(n)$	2708821	31	54	$(n)2^{18} + 69581$	23	4718592	$3(n)$	3757397	43
54	$(n)p + 1$	37	36	$3(n) + 2$	19	29	54	$(n)p + 1$	41	36	$3(n) + 2$	23	35
54	$(n)2^{2} + 2$	37	72	$3(n) + 2$	1	1	54	$(n)2^{2}$	41	72	$3(n)$	41	31
54	$(n)2^{3} + 4$	37	144	$3(n) + 1$	109	41	54	$(n)2^{3} + 6$	41	144	$3(n) + 2$	77	29
54	$(n)2^{4}$	37	288	$3(n)$	37	7	54	$(n)2^{4} + 10$	41	288	$3(n) + 2$	5	1
54	$(n)2^{5} + 24$	37	576	$3(n) + 2$	181	17	54	$(n)2^{5} + 18$	41	576	$3(n) + 1$	437	41
54	$(n)2^{6} + 40$	37	1152	$3(n) + 1$	1045	49	54	$(n)2^{6} + 2$	41	1152	$3(n)$	149	7
54	$(n)2^{7} + 8$	37	2304	$3(n)$	469	11	54	$(n)2^{7} + 98$	41	2304	$3(n) + 2$	725	17
54	$(n)2^{8} + 200$	37	4608	$3(n) + 2$	1621	19	54	$(n)2^{8} + 162$	41	4608	$3(n) + 1$	4181	49
54	$(n)2^{9} + 72$	37	9216	$3(n)$	3925	23	54	$(n)2^{9} + 34$	41	9216	$3(n)$	1877	11
54	$(n)2^{10} + 840$	37	18432	$3(n) + 2$	8533	25	54	$(n)2^{10} + 802$	41	18432	$3(n) + 2$	6485	19
54	$(n)2^{11} + 1352$	37	36864	$3(n) + 1$	36181	53	54	$(n)2^{11} + 290$	41	36864	$3(n)$	15701	23
54	$(n)2^{12} + 328$	37	73728	$3(n)$	17749	13	54	$(n)2^{12} + 3362$	41	73728	$3(n) + 2$	34133	25
54	$(n)2^{13} + 2376$	37	147456	$3(n)$	128341	47	54	$(n)2^{13} + 5410$	41	147456	$3(n) + 1$	144725	53
54	$(n)2^{14} + 14664$	37	294912	$3(n) + 2$	202069	37	54	$(n)2^{14} + 1314$	41	294912	$3(n)$	70997	13
54	$(n)2^{15} + 22856$	37	589824	$3(n) + 2$	54613	5	54	$(n)2^{15} + 9506$	41	589824	$3(n)$	513365	47
54	$(n)2^{16} + 39240$	37	1179648	$3(n) + 1$	939349	43	54	$(n)2^{16} + 58658$	41	1179648	$3(n) + 2$	808277	37
54	$(n)2^{17} + 72008$	37	2359296	$3(n) + 1$	1529173	35	54	$(n)2^{17} + 91426$	41	2359296	$3(n) + 2$	218453	5
54	$(n)2^{18} + 137544$	37	4718592	$3(n) + 1$	2708821	31	54	$(n)2^{18} + 156962$	41	4718592	$3(n) + 1$	3757397	43

Fig.6. A partial result that demonstrates how certain partitions of $[e]_{54}$ are related to known partitions of $[d]_{36}$ numbers, establishing an associated instantiation of certain next odd integers to further Collatz subsequences, e.g. *54(4n)+1 = 72(3n)+1, 54(4n+2)+5 = 72(3n+1)+41, 54(8n+2)+1 = 144(3n)+109, etc.*

Such further analyses (as demonstrated in Fig. 6) have induced us to construct multilevel-based dynamics between the *next odd* numbers and the immediate (further) subsequent *next odd* numbers. This has facilitated the construction of a general multilevel number-dynamics framework that characterizes and represents the Collatz dynamical system of odd-to-odd number transitions. For example, the Collatz transformation map in Fig. 7 is a Collatz transformation map inferred from the general multilevel framework, the prime model in Fig. 8 is completely deterministic and derivable from the number classification layout previously presented in Fig. 3, and so on.

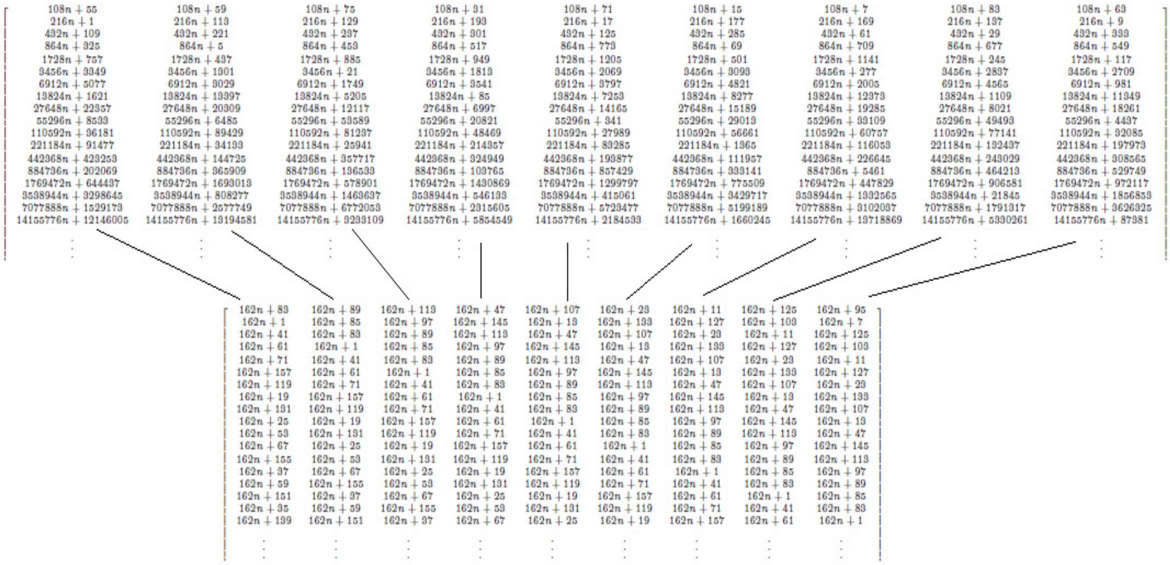

Fig.7. A subset of the general multilevel number-dynamics characterizing the Collatz system of odd-to-odd number transitions.

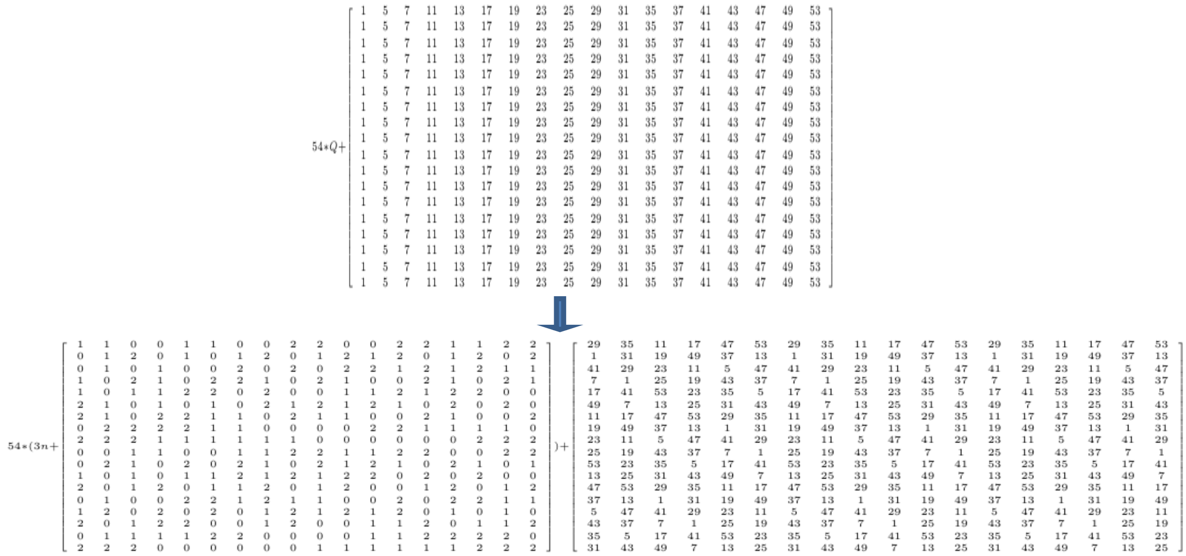

Fig.8. A prime model for the Collatz dynamical System which embeds elementary parameters (integer constants) based on the transformation matrix Q that is dependent on an integer variable *n*. Note: the matrix Q is inferable from the framework in Fig.3. Alternatively, the matrix Q is a multiplicative factor of the (top) matrix presented in Fig.7.

3. Concluding Remarks

We have presented some new results (in Fig. 3, 5, 7 and 8), presented in [18] and established here for the first time, which represent an optimized mapping and further mapping of odd integer inputs to their *next odd* integers in the Collatz-based number system. The principal result presented in Fig. 6 evidences a new method of instantiating (i.e. identifying and casting) between equivalent (i.e. in magnitude) $[e]_{54}$ and $[d]_{36}$ numbers. This has resulted in a robust and tractable method for representing the stopping time(s) of odd integers within the Collatz sequences of transformation. The observed phenomena in Fig. 7, in addition to facilitating a multilevel mapping of some certain odd-to-odd mappings, also demonstrates an example of an application of a multilevel characterization and potential of the proposed *prime model* (Fig. 8) for the Collatz dynamical system. Other *hidden* integer-dynamics are currently being uncovered through the proposed strategy which are introducing and generating new results and insights into some of the mathematical mysteries that characterize the Collatz conjecture or 3n+1 challenge. For example, in Fig. 6 it could be verified that $54(8n+2)+1 = 144(3n)+109$. Such insightful mappings are currently being used to investigate and understand the Collatz dynamical system (principally from $4n+3$ inputs) in order to produce an irrefutable proof of the (un)decidability of the $3n+1$ conjecture.

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